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Limitations and Strengths of the Fourier Transform Method to Detect Accelerating Targets

T. Thayaparan
Defence Research Establishment Ottawa

A. Yasotharan
Unique Broadband Systems

Defence R&D Canada

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T. Thayaparan
Ground Based Radar Group
Surface Radar Section

A. Yasotharan
Unique Broadband Systems

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Abstract

There are limitations and shortcomings to the Fourier transform method to detect accelerating targets because of the phenomenon known as Doppler smearing. In using a Pulse Doppler Radar to detect a non-accelerating target in additive white Gaussian noise and to estimate its radial velocity, the Fourier method provides an output signal-to-noise ratio (SNR) that increases linearly with the number of pulses. When the target is accelerating, the Fourier method may still be used to detect the target and estimate its median velocity, provided the acceleration is small enough in the sense described in the paper. For a given acceleration, when the number of pulses is increased, the output SNR of the Fourier method varies as a concave function, increasing to a maximum and then decreasing, before the method fails catastrophically. Thus the number of pulses and the acceleration have to be matched to achieve optimum performance. Empirical formulae for the dependence of the optimum SNR and the optimum number of pulses on the acceleration are given. The results are shown to be relevant to the design of Generalized Likelihood Ratio Test (GLRT) based detectors that apply a search over a grid.

Résumé

Ils y des limites et des problèmes associés avec l'utilisation de la méthode de transformdée de Fourier pour la détection des cibles qui accélèrent à cause du phénomène d'élargissement Doppler. Dans l'utilisation des radars Doppler à impulsions pour la détection de cibles non accélérante en présence de bruit blanc et l'estimation de la vélocité radiale, la méthode de transformée de Fourier fournit un rapport de signal au bruit (SNR) qui augmente linéairement avec le nombre de pulses. Quand la cible est en train d'accélérer la méthode de Fourier peut être encore utilisée pour détecter la cible et sa vitesse médiane en autant que l'accélération soit suffisamment petite comme montrée dans ce rapport. Pour une accélération donnée, quand le nombre de pulse augmente, le SNR résultant de la transformée de Fourier varie comme une fonction concave, augmentant á un maximum et après décroissant, avant que la méthode échoue de façon catastrophique. Ainsi le nombre de pulse et l'accélération de la cible doivent être relier ensemble pour accomplir une performance optimum. Des formules empiriques donnant le SNR optimum en fonction du nombre de pulse et l'accélération sont données. Les résultats sont relevant au développement de détecteurs utilisant un test de rapport de probabilité généralisé qui applique une recherche a travers une grille.

Executive Summary

The Fourier Transform is at the heart of a wide range of techniques that are used in HF radar data analysis and processing. Mapping the data into the temporal frequency domain is an effective way of recording the data such that their global characteristics can be assessed. However, the change of frequency content with time is one of the main features we observe in HF radar data. Because of this change of frequency content with time, radar signals belong to the class of non-stationary signals.

One of the central problems in High Frequency (HF) radar data is the analysis of a time series. The Fourier transform method, or Doppler processing method, has been generally used in HF radar to detect targets that are moving with constant radial acceleration. Examples of accelerating targets are manoeuvring aircrafts and missiles. In this report we show that there are limitations and shortcomings to the Fourier transform method to detect accelerating targets because of the phenomenon known as Doppler smearing. We show that when the target is constantly accelerating, the Fourier method may still be used to detect target and estimate its median velocity, provided the acceleration is small enough in the sense to be described in this report. It is shown that for a given acceleration, the number of pulses cannot be increased indefinitely without resulting in catastrophic failure of the method. Conversely, for a given number of pulses, the acceleration cannot be arbitrarily large without resulting in catastrophic failure of the method. Thus the number of pulses and the acceleration have to be matched to achieve optimum performance.

Author, Thayananthan, Thayaparan, Limitations and strengths of the Fourier transform method to detect accelerating targets, Defence Research Establishment Ottawa, DREO TM 2000-078, November 2000

Sommaire

La transformée de Fourier est au cœur d'un large éventail de techniques qui sont utilisées dans l'analyse et le traitement des données des radars à hautes fréquences. Le mappage des données dans le domaine temporel de fréquence est une façon effective d'enregistrer les données tel que leurs caractéristiques globales peuvent être évaluer. Cependant, le changement des composantes spectrales avec le temps est une des caractéristiques que nous observons dans les données radar à haute fréquence. Ces signaux radars appartiennent à la classe des signaux non stationnaires à cause de ce changement de composantes spectrales avec le temps.

Un des problèmes centraux dans les données des radars à haute fréquence est l'analyses des séries temporelles. La méthode des transforme de Fourier ou la méthode de traitement Doppler a été généralement utilisée dans les radars à haute fréquence pour détecter des cibles qui se déplace avec une accélération radiale constante. Exemples de cible qui accélèrent sont des avions et des missiles qui manœuvrent. Dans ce rapport, nous montrons qu'ils y des limites et des problèmes associés avec l'utilisation de la méthode de transformée de Fourier pour la détection des cibles qui accélèrent à cause du phénomène d'élargissement Doppler. Nous montrons que quand la cible est en train d'accélérer la méthode de Fourier peut être encore utilisée pour détecter la cible et sa vitesse médiane en autant que l'accélération soit suffisamment petite comme montrée dans ce rapport. Ce rapport montre que pour une accélération donne, le nombre de pulse ne peut être augmenter indéfiniment sans que la méthode échoue de façon catastrophique. D'un autre cotée, pour un nombre donne de pulses, l'accélération ne peut être infiniment grand sans que la méthode échoue de façon catastrophique. Ainsi le nombre de pulse et l'accélération doivent être relier ensemble pour accomplir une performance optimum.

Auther, Thayananthan, Thayaparan, Limitations et points forts de la méthode de transformée de Fourier pour la détection de cibles qui accélère, Center de Recherches pour la Défense, Ottawa, DREO TM 2000-078, Novembre 2000

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1 Introduction

The Fourier Transform is at the heart of a wide range of techniques that are used in HF radar data analysis and processing. Mapping the data into the temporal frequency domain is an effective way of recording the data such that their global characteristics can be assessed. However, the change of frequency content with time is one of the main features we observe in HF radar data. Because of this change of frequency content with time, radar signals belong to the class of non-stationary signals. One of the central problems in High Frequency (HF) radar data is the analysis of a time series. The Fourier transform method, or Doppler processing method, has been generally used in HF radar to detect targets that are moving with constant radial acceleration. Examples of accelerating targets are manoeuvring aircrafts and missiles.

In this paper, the radar is assumed to be of the Pulse-Doppler type, that is, it sends out a uniform train of radio frequency pulses and phase-coherently receives their returns. It is also assumed that the receiver has pulse compression capability so that a pulse return from an isolated target can be represented by a single sample of the compressed pulse.

A moving target with low per-pulse signal-to-noise ratio (SNR) may be efficiently detected by processing the sequence of Pulse-Doppler radar returns by coherent integration [*Whalen*, 1971: *Meyer and Mayer*, 1973; *Wehner*, 1987; *Scheer*, 1993]. Under the simplifying assumptions of zero range-walk and constant radial velocity, coherent integration is achieved by taking the Fourier power spectrum of the sequence of complex amplitudes of the received pulses and locating its maximum. If the maximum value exceeds a threshold then the presence of a target is declared, and the location of the maximum is taken to be an estimate of the radial velocity. In an additive white noise background, the Fourier method gives an output SNR that is proportional to the number of pulses [*Meyer and Mayer*, 1973].

In this paper, we relax the assumption of constant radial velocity and consider using the Fourier method to detect targets that are moving with constant radial acceleration. Thus the aim of the paper is to study the phenomenon usually referred to as *Doppler smearing* and quantify the performance of the Fourier method. In this paper, the term *acceleration* may be interpreted as the *magnitude of the acceleration*, as the conclusions depend only on the magnitude of the acceleration. It is empirically shown that when the target is constantly accelerating, the Fourier method may still be used to detect the target and estimate its

median velocity, provided the acceleration is small enough in the sense to be described in this paper.

It is shown that for a given acceleration, the number of pulses cannot be increased indefinitely without resulting in catastrophic failure of the method. Conversely, for a given number of pulses, the acceleration cannot be arbitrarily large without resulting in catastrophic failure of the method. Moreover, for a given acceleration, when the number of pulses is increased, the output SNR varies as a concave function of the number of pulses, achieving a maximum well before the failure occurs. For a given number of pulses, the ouput SNR monotonically decreases with the acceleration, until the failure occcurs. Thus the number of pulses and the acceleration have to be matched to achieve optimum performance.

The results are presented in terms of a normalized value of the acceleration that takes into account the radar carrier frequency and the pulse repetition freqency. This makes the results widely useful. The results may be used to determine whether, in a given scenario of an accelerating target, the Fourier method can be used or a more sophisticated method such as the Generalized Likelihood Ratio Test (GLRT) is required. In scenarios where the Fourier method is not desirable, and where the GLRT method is considered, it is shown how the results may be used to determine the critical spacing of the grid over which the search is applied.

2 Signal Model

Suppose the Pulse-Doppler radar sends out N pulses, one every T seconds, and there is a target moving with constant radial acceleration. Assume that the change in range during the observation period of NT seconds, known as range walk, is negligible compared to the radar range resolution as determined by the width of the compressed pulse. Then the complex amplitudes of the N range-compressed pulses taken at the range of the target have the form

$$r(n) = s(n) + v(n) \tag{1}$$

where

$$s(n) = ae^{j(b_0 + b_1 n + \frac{1}{2} b_2 n^2)} \tag{2}$$

for $n = 0, 1, 2, 3, \ldots, (N-1)$, is the noise-free signal, and $v(n)$ is a sequence of independent samples of complex Gaussian noise with mean zero and variance σ^2.

The signal parameters a and b_0 are the target signal amplitude and phase respectively. The signal parameters b_1 and b_2 are the normalized initial radial velocity and the normalized radial acceleration respectively.

2.1 Normalized Initial Radial Velocity and Radial Acceleration

The normalized initial radial velocity b_1 is defined as

$$b_1 = u\left(\frac{4\pi T}{\lambda}\right). \tag{3}$$

where u is the initial radial velocity in meters/sec towards the radar, T is pulse repetition interval in secs, and λ is the carrier wavelength in meters. Similarly, the normalized radial acceleration b_2 is defined as

$$b_2 = f\left(\frac{4\pi T^2}{\lambda}\right). \tag{4}$$

where f is the radial acceleration in meters/sec/sec towards the radar. It can be seen that both of the above normalized quantities are non-dimensional.

To obtain these relations note that the decrement in range d, measured from the beginning of the observation interval, as a function of continuous-time t, is $d(t) = ut + \frac{1}{2}ft^2$, and therefore the phase increment of a pulse as a function of discrete-time, or pulse index, is given by

$$2\pi\left(\frac{2d(nT)}{\lambda}\right) = u\left(\frac{4\pi T}{\lambda}\right)n + \frac{1}{2}f\left(\frac{4\pi T^2}{\lambda}\right)n^2. \tag{5}$$

In the rest of the discussion, the terms *velocity* and *acceleration* refer to *normalized radial velocity* and *normalized radial acceleration respectively.*

2.2 Per-Pulse Signal-to-Noise Ratio

The Per-Pulse Signal-to-Noise Ratio is defined as

$$\mathrm{SNR}_{\mathrm{pulse}} = \left(\frac{a}{\sigma}\right)^2. \tag{6}$$

3 The performance of the Fourier method in detecting a non-accelerating target

Although a given signal can be represented in many different ways, the most important are the time and frequency representations. The majority of signals encountered in our everyday life are directly related to time. The frequency representations, on the other hand, were not popular until the early 19th century when Fourier first proposed the harmonic trigonometric series. Since then, the frequency representation has become one of the most powerful and standard tools for studying signals. By using frequency representations, we could better understand many physical phenomenon and accomplish many things that cannot achieved based on time representations. While the time domain function indicates how a signal's amplitude changes with time, the frequency domain function tells how often such changes take place. The bridge between time and frequency is the *Fourier Transform*.

The signal is expanded in terms of sinusoids of different frequencies [*Brigham*, 1974; *Bloomfield*, 1976; *Papoulis* 1977; *Bracewell*, 1978]

$$s(t) = \frac{1}{\sqrt{2\pi}} \int S(\omega)\, e^{j\omega t}\, d\omega \tag{7}$$

The waveform is made up of the addition (linear superposition) of the simple waveforms, $e^{j\omega t}$, each characterized by the frequency, ω, and contributing a relative amount indicated by the coefficient, $S(\omega)$. $S(\omega)$ is obtained from the signal by

$$S(\omega) = \frac{1}{\sqrt{2\pi}} \int s(t)\, e^{-j\omega t}\, dt \tag{8}$$

and is called the spectrum or the Fourier Transform. The square of the Fourier Transform $| S(\omega) |^2$ is called the *Fourier power spectrum*, which indicates how the signal energy is distributed in the frequency domain. While the Fourier Transform $S(\omega)$ is a linear function of the analyzed signal, the Fourier power spectrum $| S(\omega) |^2$ is quadratic to the signal $s(t)$. The Fourier Transform $S(\omega)$ in general is complex, whereas the power spectrum $| S(\omega) |^2$ is always real. The Fourier Transform and the Fourier power spectrum are the two most important tools for frequency analysis.

Before investigating the case of an accelerating target, it is helpful to review the case of a

non-accelerating target.

When the acceleration is zero, and the noise is absent, the Fourier power spectrum

$$S(\omega) = \frac{1}{N}\left|\sum_{n=0}^{N-1} s(n)e^{-j\omega n}\right|^2 \tag{9}$$

$$= \frac{a^2}{N}\left|\sum_{n=0}^{N-1} e^{j(b_1-\omega)n}\right|^2, \tag{10}$$

has a unique maximum which is attained when $\omega = b_1 \bmod 2\pi$. Assuming $-\pi < b_1 < \pi$, the maximizing ω is an estimate of the normalized radial velocity.

Therefore, the target can be detected and the velocity parameter b_1 can be estimated by using peak detection on the noisy version

$$\Gamma(\omega) = \frac{1}{N}\left|\sum_{n=0}^{N-1} r(n)e^{-j\omega n}\right|^2 \tag{11}$$

of $S(\omega)$, that is, maximizing $\Gamma(\omega)$ with respect to ω, and declaring the presence of a target if the maximum value exceeds a threshold; the maximizing ω, which is now a random variable due to the presence of noise, is taken as the estimate of b_1. This method is equivalent to the GLRT.

The notion of output SNR of the method comes from the observation that under the 'noise-only' hypothesis $\frac{2}{\sigma^2}\Gamma(\omega)$ is a central Chi-Squared random variable of two degrees of freedom, for all ω, whereas under the 'signal-and-noise' hypothesis $\frac{2}{\sigma^2}\Gamma(b_1)$ is a non-central Chi-Squared random variable of two degrees of freedom and non-centrality parameter $2N\left(\frac{a}{\sigma}\right)^2$ [*Whalen*, 1971; *Meyer and Mayer*, 1973].

We, therefore, consider

$$\mathrm{SNR}_{\mathrm{out}} = N\left(\frac{a}{\sigma}\right)^2, \tag{12}$$

as this can be used to approximately compute the probability of detection, assuming that the maximum of $\Gamma(\omega)$ occurs at $\omega = b_1$. Note that $\mathrm{SNR}_{\mathrm{out}}$ is a monotonically increasing

function of N. More specifically, it is a linearly increasing function of N. Normalizing this with respect to $\mathrm{SNR}_{\mathrm{pulse}}$ gives

$$\mathrm{SNR}_{\mathrm{gain}} = \frac{\mathrm{SNR}_{\mathrm{out}}}{\mathrm{SNR}_{\mathrm{pulse}}}, \tag{13}$$

$$= N. \tag{14}$$

4 The performance of the Fourier method in detecting an accelerating target

When the acceleration is non-zero, and the noise is absent, the Fourier power spectrum is

$$S(\omega) = \frac{1}{N}\left|\sum_{n=0}^{N-1} s(n)e^{-\jmath\omega n}\right|^2, \tag{15}$$

$$= \frac{a^2}{N}\left|\sum_{n=0}^{N-1} e^{\jmath\left((b_1-\omega)n+\frac{1}{2}b_2n^2\right)}\right|^2. \tag{16}$$

Some insight into this function can be gained by rearranging it as

$$S(\omega) = \frac{a^2}{N}\left|\sum_{n=0}^{N-1} e^{\jmath\left((b_m-\omega)\left(n-\frac{N-1}{2}\right)+\frac{1}{2}b_2\left(n-\frac{N-1}{2}\right)^2\right)}\right|^2, \tag{17}$$

where

$$b_m = b_1 + \left(\frac{N-1}{2}\right)b_2 \tag{18}$$

is the normalized median velocity over the time interval 0 to $(N-1)T$ seconds. For simplicity of discussion assume that $-\pi < b_m < \pi$. The rearrangement shows that $S(\omega)$ is symmetric with respect to $\omega = b_m$, that is, $S(b_m+\nu) = S(b_m-\nu)$, and the spread of the function about $\omega = b_m$ depends only on b_2 and N. More specifically, when considered as a function of b_2, $S(\omega)$ depends only on the magnitude of b_2.

A closed-form expression for the above sum seems difficult to obtain. But numerical evaluations suggest that, for a given acceleration b_2, there is a threshold value N_t of N, below which $S(\omega)$ has a unique maximum, and above which $S(\omega)$ has at least two equal maxima. The threshold value N_t depends on the acceleration b_2 alone. Moreover, N_t is a decreasing function of b_2. Thus N_t implicitly defines the maximum allowable acceleration for a given N.

4.1 The Region $N < N_t$

For $N < N_t$, by symmetry considerations, the unique maximum of $S(\omega)$ is attained when $\omega = b_m$. Thus

$$\max_{\omega} S(\omega) = S(b_m), \tag{19}$$

$$= \frac{a^2}{N}\left|\sum_{n=0}^{N-1} e^{j\frac{1}{2}b_2\left(n-\frac{N-1}{2}\right)^2}\right|^2. \tag{20}$$

Therefore, in this region, we have

$$\mathrm{SNR}_{\mathrm{out}} = \frac{S(b_m)}{\sigma^2}, \tag{21}$$

and

$$\mathrm{SNR}_{\mathrm{gain}} = \frac{\mathrm{SNR}_{\mathrm{out}}}{\mathrm{SNR}_{\mathrm{pulse}}}, \tag{22}$$

$$= \frac{S(b_m)}{a^2}, \tag{23}$$

$$= \frac{1}{N}\left|\sum_{n=0}^{N-1} e^{j\frac{1}{2}b_2\left(n-\frac{N-1}{2}\right)^2}\right|^2. \tag{24}$$

Provided that $\mathrm{SNR}_{\mathrm{out}}$ is adequate, peak detection still provides a means of detecting an accelerating target and estimating its median velocity. However, as expected, $\mathrm{SNR}_{\mathrm{out}}$ is not a linearly increasing function of N. It is not even a monotonically increasing function of N. Rather it is a concave function of N and attains a maximum value in the region $N < N_t$. The optimum value N_{opt} of N, where the $\mathrm{SNR}_{\mathrm{out}}$ is maximum, depends on the parameter b_2 alone. Moreover, N_{opt} is a decreasing function of b_2.

For $N < N_{opt}$, the concentration of the Fourier power spectrum $S(\omega)$ about $\omega = b_m$ increases as N increases towards N_{opt}. For $N > N_{opt}$, the concentration of the Fourier power spectrum $S(\omega)$ about $\omega = b_m$ decreases as N increases towards N_t.

4.2 The Region $N > N_t$

For $N > N_t$, the Fourier power spectrum $S(\omega)$ has at least two maxima located symmetrically about $\omega = b_m$. Moreover, it is spread symmetrically about $\omega = b_m$, and the amount of spread

increases rapidly with N, soon covering the entire interval of length 2π. The combination of the significantly low maxima and the spectral spreading makes it practically impossible to discern the presence of the target or estimate its spectral location. Therefore, the use of peak detection as a method of detecting an accelerating target and estimating its median velocity is precluded in this region.

5 Numerical Examples

Figure 1 shows how the Fourier spectrum changes when N (referred to as the length) is increased while the acceleration is held constant. Here we can see the spectrum concentration first increasing, then slowly decreasing, and eventually rapidly decreasing with multiple peaks appearing.

Figure 2 shows how the Fourier spectrum changes when the acceleration is increased while N is held constant. Here we can see the spectrum concentration first slowly decreasing and then rapidly decreasing with multiple peaks appearing.

In all cases the median velocity was held at zero for the purpose of compact representation, but the conclusions are independent of this assumption.

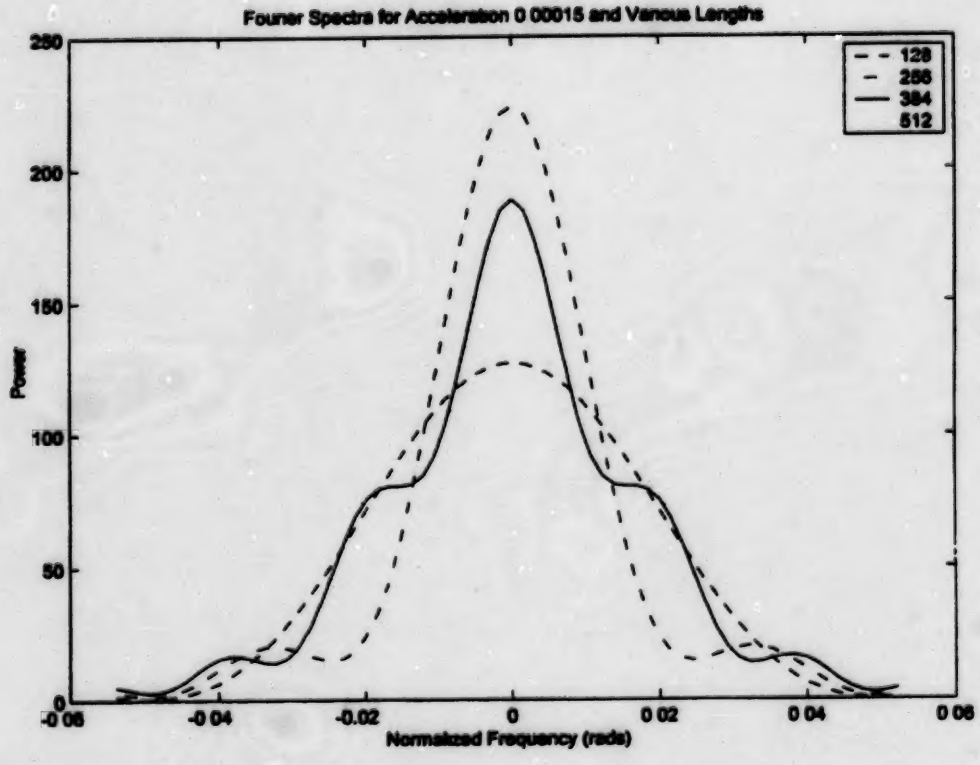

Figure 1: Effect of increasing length.

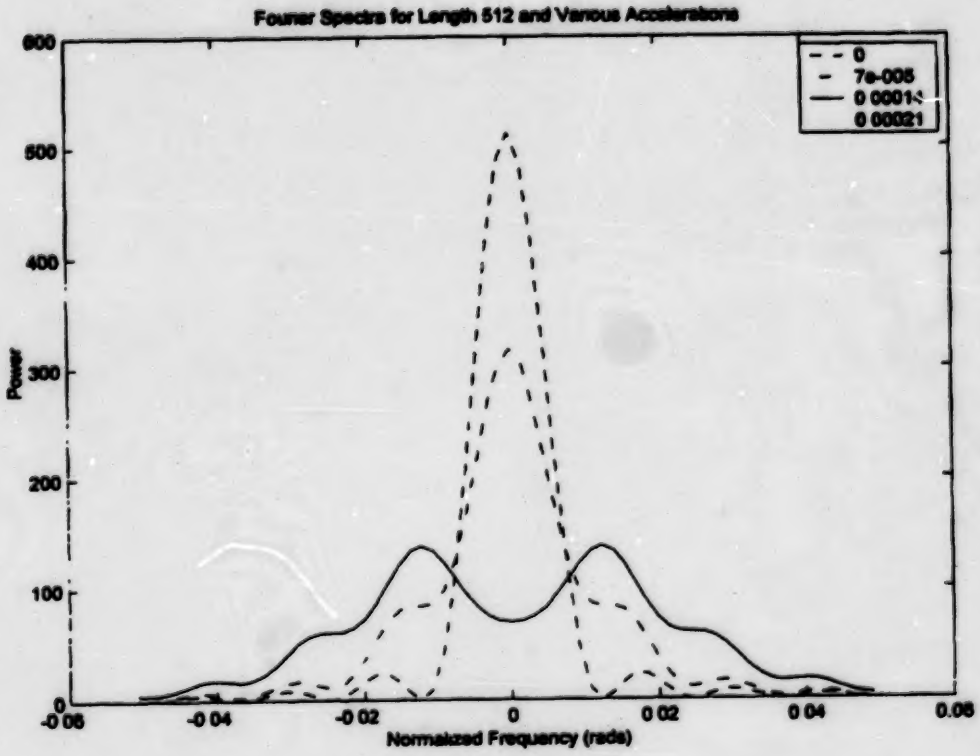

Figure 2: Effect of increasing acceleartion.

6 Performance Graphs

Graphs quantifying the performance of the Fourier method in detecting an accelerating target are given in this section. The graphs were obtained by empirically studying the Fourier power spectrum for values of N ranging from 64 to 512 and acceleration b_2 ranging from 0.00005 to 0.0004.

Figures 3 and 4 show the dependence of the SNR_{gain} on Length N of the signal for various values of the acceleration b_2. We can see that for $b_2 \geq 0.00012$, the failure point has been achieved, that is, $N_t < 512$, and for $b_2 \geq 0.00006$, the optimum point has been achieved, that is $N_{opt} < 512$.

Figure 5 shows the dependence of the Optimum SNR_{gain} on acceleration. Figure 6 shows the dependence of the Optimum Length N_{opt} on acceleration.

Figure 7 shows the dependence of the log of Optimum SNR_{gain} on the log of acceleration. From the linear least squares fit we can infer that

$$10 \log_{10} \text{SNR}_{\text{gain,opt}} \approx -5 \log_{10} b_2 + 4.537. \tag{25}$$

or

$$\text{SNR}_{\text{gain,opt}} \approx 10^{0.4537} b_2^{-0.5}. \tag{26}$$

Figure 8 shows the dependence of the log of optimum length on the log of acceleration. From the linear least squares fit we can infer that

$$\log_{10} N_{opt} \approx -0.5 \log_{10} b_2 + 0.5606, \tag{27}$$

or

$$N_{opt} \approx 10^{0.5606} b_2^{-0.5}. \tag{28}$$

Combining these,

$$\mathrm{SNR}_{\mathrm{gain,opt}} \approx 10^{-0.112} N_{opt}. \tag{29}$$

Thus when operating at the optimum point $N = N_{opt}$, the loss with respect to the non-accelerating case is 1.12dB. This is also the loss with respect to optimum coherent integration.

Every effort was made to ensure the accuracy of the above results. However, caution must be exercised in applying them outside the studied range, especially for larger accelerations.

Figure 9 shows the dependence of the log of threshold $\mathrm{SNR}_{\mathrm{gain}}$ on the log of acceleration. From the linear least squares fit we can infer that

$$10 \log_{10} \mathrm{SNR}_{\mathrm{gain,opt}} \approx -4.677 \log_{10} b_2 + 3.746, \tag{30}$$

or

$$\mathrm{SNR}_{\mathrm{gain,opt}} \approx 10^{0.3746} b_2^{-0.4677}. \tag{31}$$

Figure 10 shows the dependence of the log of threshold length on the log of acceleration. From the linear least squares fit we can infer that

$$\log_{10} N_t \approx -0.5078 \log_{10} b_2 + 0.6859, \tag{32}$$

or

$$N_t \approx 10^{0.6859} b_2^{-0.5078}. \tag{33}$$

Combining these,

$$\mathrm{SNR}_{\mathrm{gain,opt}} \approx 10^{-0.3745} N_t^{0.9210}. \tag{34}$$

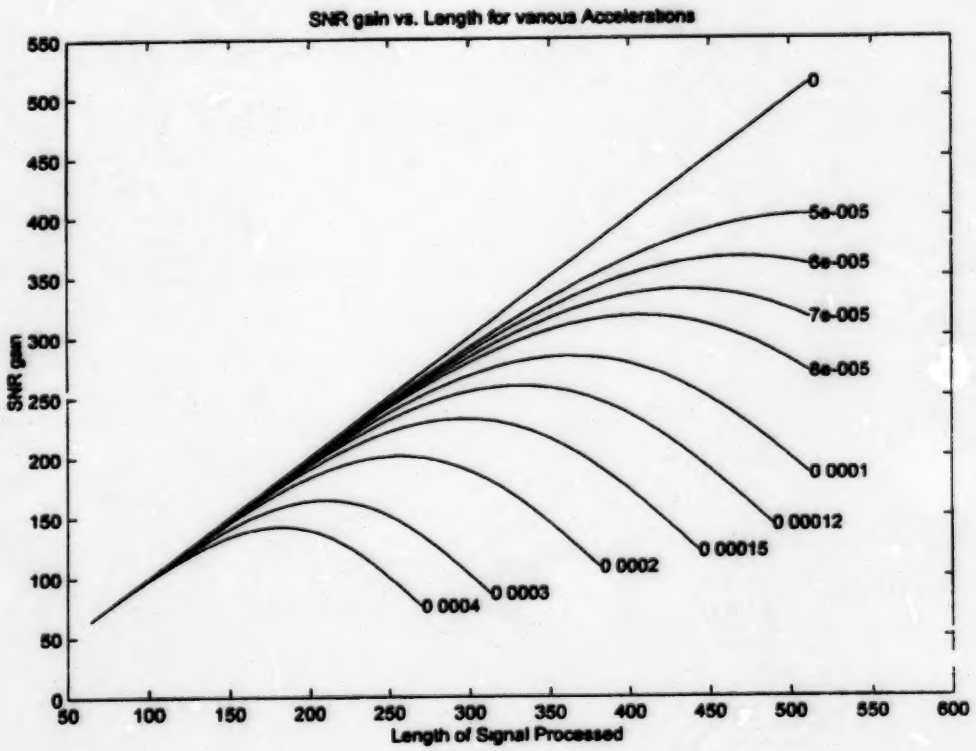

Figure 3: The dependence of SNR on length for various accelerations.

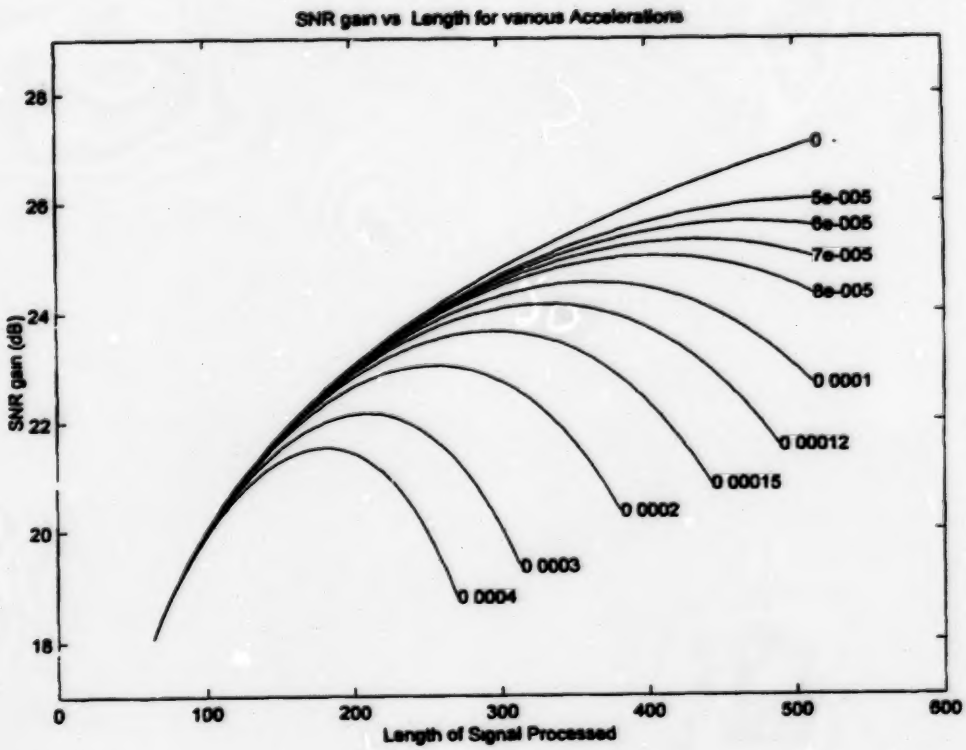

Figure 4: The dependence of SNR on length for various accelerations.

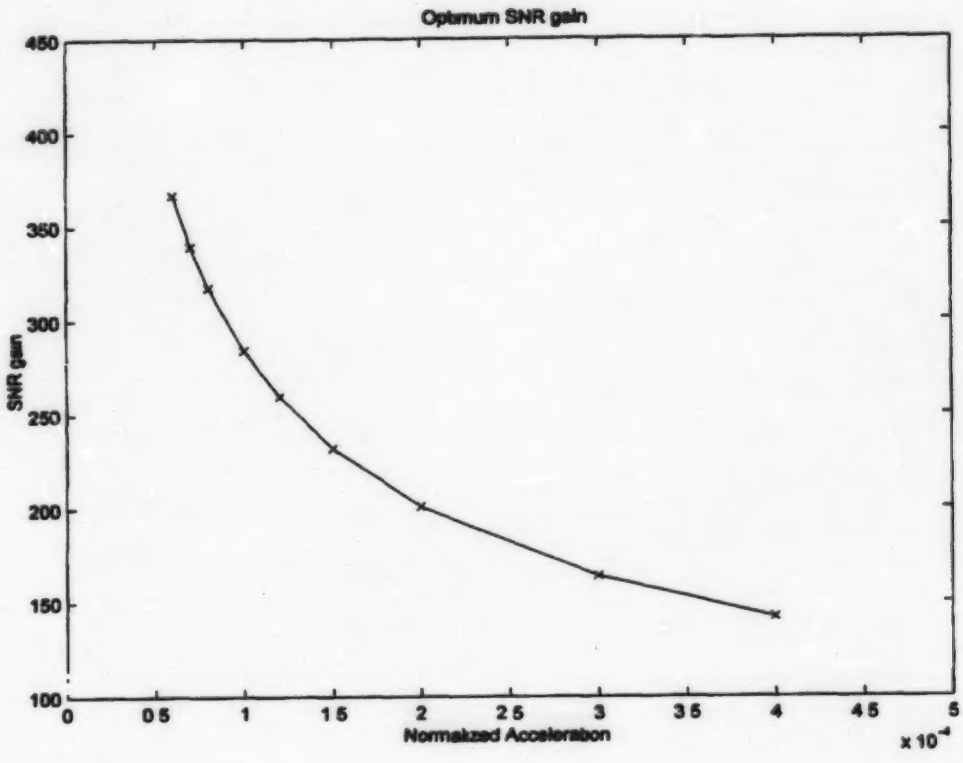

Figure 5: The dependence of the optimum SNR on the accelerations.

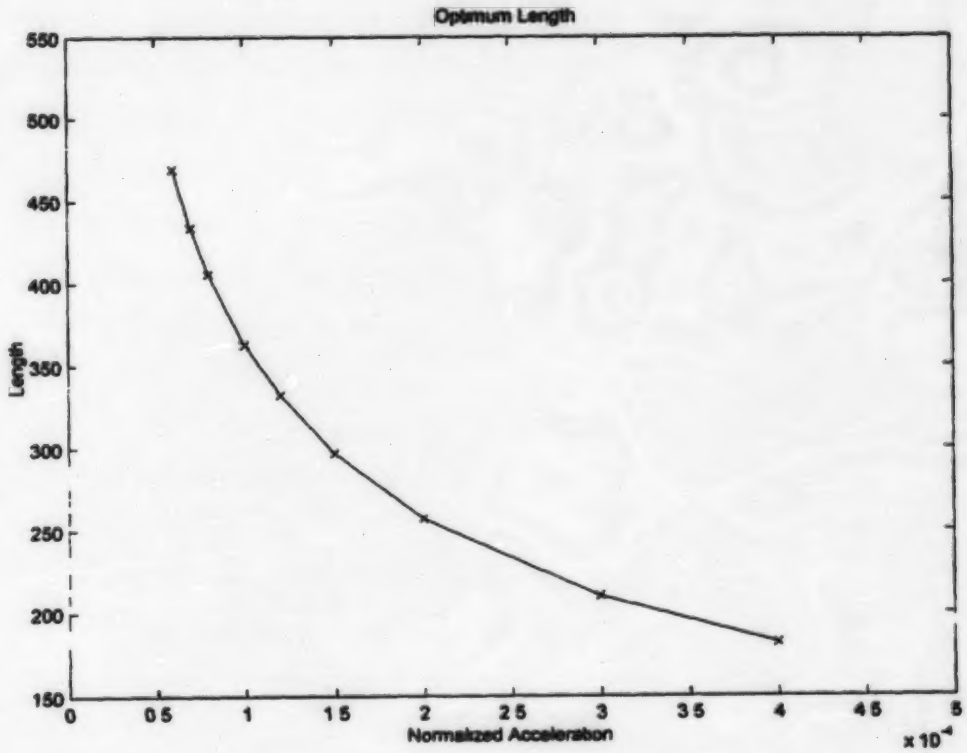

Figure 6: The dependence of the optimum length on the accelerations.

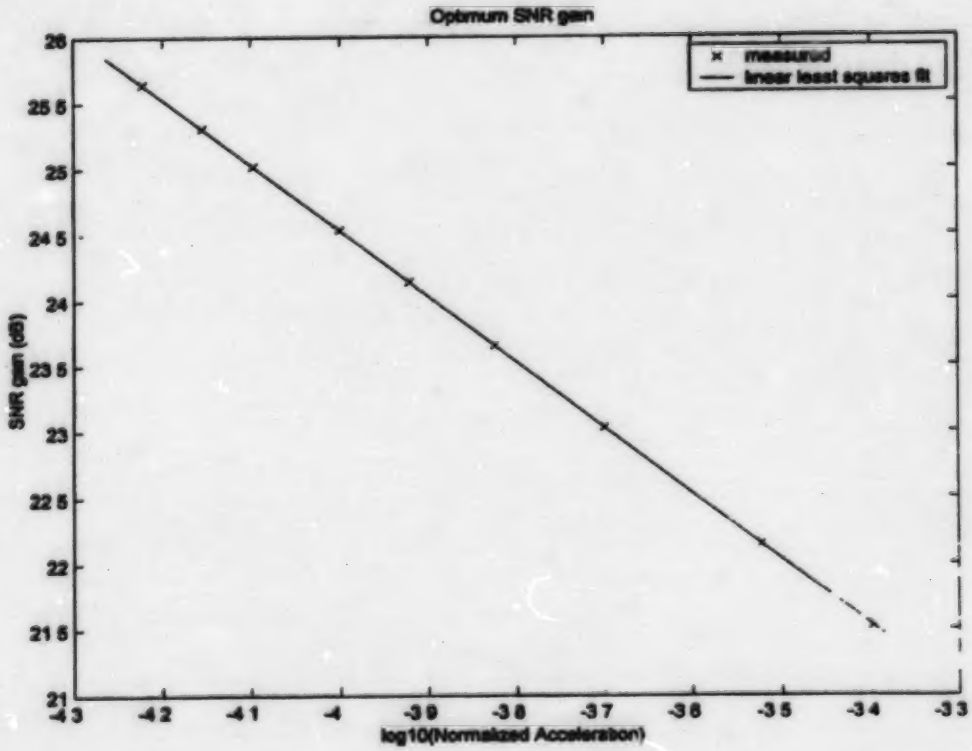

Figure 7: The dependence of the log10(optimum SNR) on the log10(acceleration).

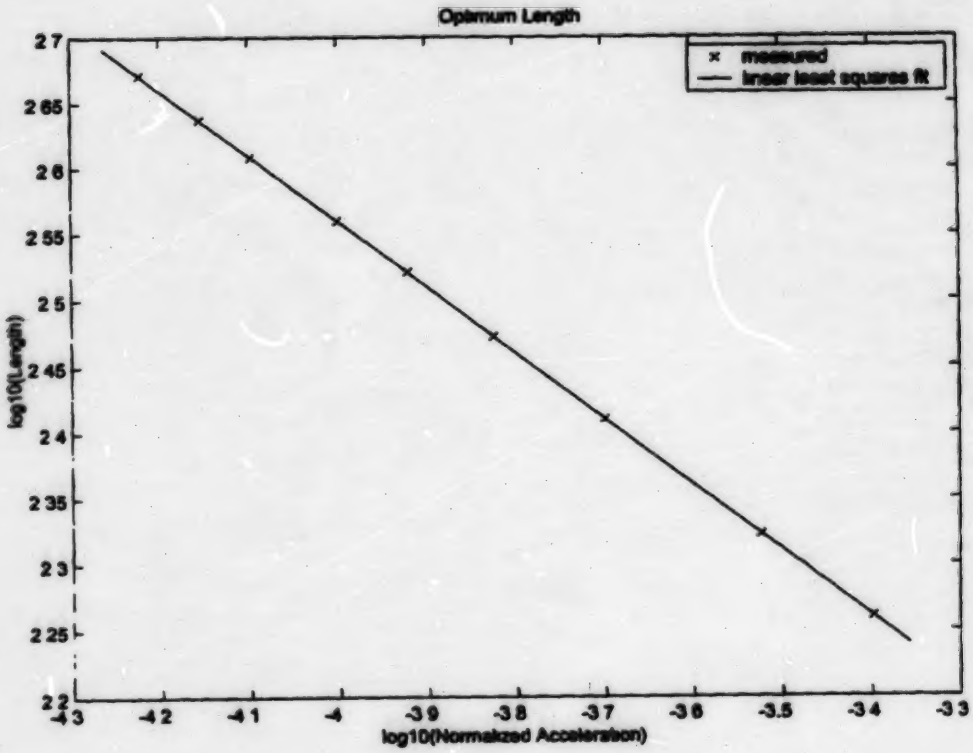

Figure 8: The dependence of the log10(optimum length) on the log10(acceleration).

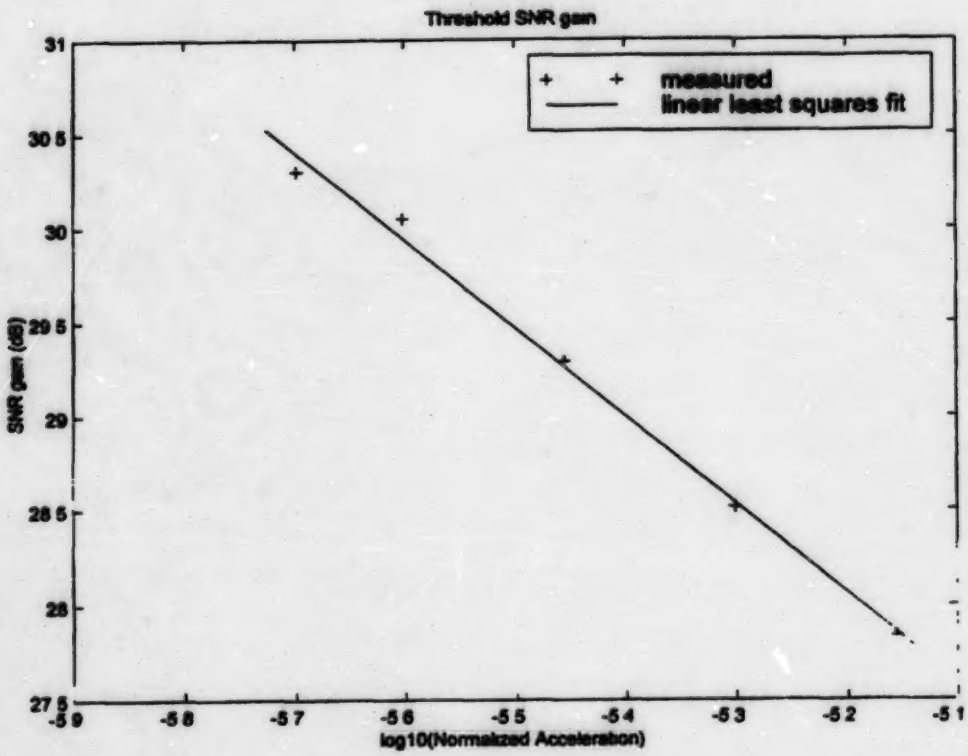

Figure 9: The dependence of the log10(threshold SNR) on the log10(acceleration).

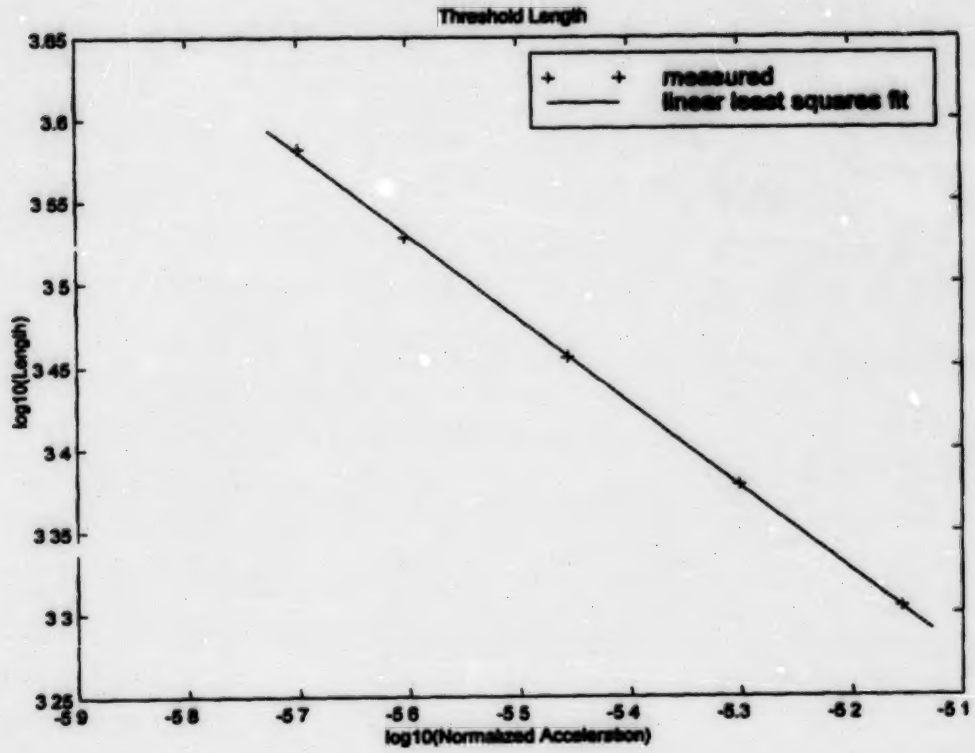

Figure 10: The dependence of the log10(threshold length) on the log10(acceleration).

7 Application to GLRT

In the case of an accelerating target, the GLRT method seeks to maximize $\Lambda(c_1, c_2) = \frac{1}{N}\left|\sum_{n=0}^{N-1} r(n)e^{-j(c_1 n + \frac{1}{2}c_2 n^2)}\right|^2$ with respect to c_1 and c_2 [*Whalen*, 1971; *Kay*, 1993]. A crude way of doing this is to evaluate $\Lambda(c_1, c_2)$ on a rectangular grid in the c_1-c_2 plane and choose the maximum. The results of the previous section imply that the grid spacing in the c_2 dimension cannot be less than a critical value if the method is to work for all values of b_2.

To see this, first note that, for a fixed c_2, maximizing $\Lambda(c_1, c_2)$ with respect to c_1 is equivalent to using the Fourier method to detect a target with initial velocity b_1 and acceleration $(b_2 - c_2)$. Suppose the grid spacing in the c_2 dimension is 2Δ. Then for every b_2 there is a c_2 grid value such that $|b_2 - c_2| \leq \Delta$. For this fixed c_2 value, the noise-free version of $\Lambda(c_1, c_2)$ will have a unique maximum with respect to c_1 provided the threshold length N_t corresponding to acceleration $|b_2 - c_2|$ is greater than N. To ensure uniqueness of the maximum for every b_2, we require $\Delta \leq \Delta_{\text{crit}}$, where Δ_{crit} is the acceleration for which the given N is the threshold length.

The SNR_{gain} is at most N, and depends on $|b_2 - c_2|$. We can use the graphs to choose Δ so that the loss of SNR_{gain} is less than a given value.

8 Conclusions

The Fourier transform method, or Doppler processing method, has been generally used in HF radar to detect targets that are moving with constant radial acceleration. Examples of accelerating targets are manoeuvring aircrafts and missiles. In this report we show that there are limitations and shortcomings to the Fourier transform method to detect accelerating targets because of the phenomenon known as Doppler smearing.

We considered using the Fourier method for detecting constantly accelerating targets and estimating their median velocities, and empirically showed that there are decreasing functions $N_{opt}(b_2)$ and $N_t(b_2)$ of the acceleration b_2, termed the optimum length and the threshold length, respectively, with the following properties:

1. in the region $N < N_t$ the Fourier power spectrum of the noise-free signal has a unique maximum at the median velocity, and outside of which it has at least two maxima; thus N_t implicitly defines the maximum allowable acceleration for a given N.

2. in the region $N < N_t$ the $\mathrm{SNR}_{\mathrm{gain}}$ varies as a concave function of N achieving a maximum at $N = N_{opt}$; moreover $N_{opt} < N_t$.

Formulae were given for N_{opt} and corresponding SNR in terms of the acceleration. Similarly, Formulae were given for N_t and corresponding SNR in terms of the acceleration. These formulae might lead to the design of detectors that are of manageable complexity yet have near optimum performance.

As an application, we considered a GLRT based detector that applies a search over a grid. We showed that the grid spacing must be less than a critical value. We also showed how to choose the grid spacing so as to achieve a given $\mathrm{SNR}_{\mathrm{gain}}$.

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(U) There are limitations and shortcomings to the Fourier transform method to detect accelerating targets because of the phenomenon known as Doppler smearing. In using a Pulse Doppler Radar to detect a non-accelerating target in additive white Gaussian noise and to estimate its radial velocity, the Fourier method provides an output signal-to-noise ratio (SNR) that increases linearly with the number of pulses. When the target is accelerating, the Fourier method may still be used to detect the target and estimate its median velocity, provided the acceleration is small enough in the sense described in the paper. For a given acceleration, when the number of pulses is increased, the output SNR of the Fourier method varies as a concave function, increasing to a maximum and then decreasing, before the method fails catastrophically. Thus the number of pulses and the acceleration have to be matched to achieve optimum performance. Empirical formulae for the dependence of the optimum SNR and the optimum number ofpulses on the acceleration are given. The results are shown to be relevant to the design of Generalized Likelihood Ratio Test (GLRT) based detectors that apply a search over a grid.

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